The Magnetic Field Produced by Coils and Solenoids using Hall Effect Probe

Goals of this lab

- Study the spatial dependence of the magnetic field produced by a steady current
- Calibrate the Hall effect sensor in order to measure the magnetic field
- Find the constant \( \mu_0 \)

Equipment Used

- DC Power Supply
- DMM
- Flat Coil
- Solenoid Coil
- Hall Effect Probe
- 750 Interface – Data Studio

Introduction:
Ampere’s Law states that the line integral of \( \mathbf{B} \) and \( d\mathbf{l} \) over a closed path is \( \mu_0 \) times the current enclosed in that loop:

\[
\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enclosed}}
\]

You have seen the usefulness of the law in determining, without complicated integration, the magnetic field (B-field) of current distributions that exhibit a high degree of symmetry, such as a long, straight wire. All you need to do is cleverly choose a closed path for which the B-field has a constant magnitude and direction relative to the path. In the case of a long, straight wire, the closed paths that satisfy these requirements are circles that are centered on the wire. By symmetry, the B-field direction is tangent to centered circles, and the B-field magnitude is constant, so we can “pull it out of the integral”. The remaining integral is simply the circumference of the circle, which is \( 2\pi R \), so Ampere’s law simplifies to \( B \cdot 2\pi R = \mu_0 I \) which solved for B yields:

\[
B = \frac{\mu_0 I}{2\pi R}
\]

B-field magnitude for a long, straight wire

Understanding and applying Ampere’s law to more general situations requires careful thought. For example, if the line integral is zero, this does not necessarily mean there is no magnetic field present, or no currents enclosed. Rather it means that the dot product between the path and magnetic field sums to zero, and that there is no net current enclosed. Note that “up” and “down” currents through the enclosed surface must be assigned opposite signs.

In this lab you will actually sum up the contributions of \( \mathbf{B} \cdot d\mathbf{l} \) over such a path around a solenoid, to check if their sum does indeed equal \( \mu_0 \) times the current enclosed by your...
path. The magnetic field around the solenoid will be determined by a magnetic sensor; you will measure the output voltage of the sensor (which is proportional to B) using the multimeter. The current through the solenoid will be measured by the readout on the power supply. Please note that in your experiment, I must be understood as $NI_{\text{power supply}}$ since the single wire of the solenoid contributes I to the enclosed current with each winding (turn) of the solenoid:

$$\int B \cdot dl = \mu_0 I_{\text{enclosed}} = \mu_0 NI_{\text{power supply}}$$

Equation 1

Solenoids

An application of Ampere’s Law involves a solenoid (a wire coil wound on a cylinder) with:

- $N =$ number of turns of solenoid (dimensionless)
- $R =$ radius of coil (meters)
- $I_{\text{power supply}} =$ current through solenoid (amperes)
- $L =$ length of solenoid (meters).

The B-field magnitude at the center of the solenoid is calculated to be:

$$B_C = \frac{\mu_0 NI_{\text{power supply}}}{\sqrt{4R^2 + L^2}}$$

Equation 2

and at the end of the solenoid is:

$$B_E = \frac{\mu_0 NI_{\text{power supply}}}{2\sqrt{R^2 + L^2}}$$

Equation 3.

N can be determined from B (since we can measure all other parameters) at either of these points, as well as from the line integral of tangential B along any loop passing through the coil.

Hall Effect

A magnetic field can be measured with a Hall Effect sensor. In the diagram below, a current, $I$, is transmitted through a silicon semiconductor. The potential between the top and bottom points is zero until a perpendicular magnetic field is applied which exerts a force on the moving charges. If the current consists of positive charged carriers, a positive charge will accumulate at the lower end of the semiconductor. Negative carriers flowing in the opposite direction as $I$ will induce a negative charge at the lower end. Thus, the Hall Effect can distinguish the charge of the carrier! (In this figure, conventional current, i.e. positive charge carriers, is shown.) In any case, a small but measurable potential is caused by the magnetic field. If the field is reversed, so is the polarity of the induced voltage.
You will use a Hall Effect magnetic field detector to measure the magnetic field. The detector is mounted at one end of a clear plastic block that can be oriented however you like. The output is amplified and recorded by a sensitive voltmeter. Evaluation of the line integral of the B-field's parallel components for two or more closed paths through a solenoid will determine the number of solenoid turns $N$, by application of Equation 1.

In evaluation of the line integral we will approximate infinitesimal line elements $dl$ by finite elements $\Delta l$:

$$\mu_0 N I_{\text{PowerSupply}} = \oint B \cdot dl \approx \sum B \cdot \Delta l \cos(\theta) = \sum B \cdot \Delta l$$

$\Delta l$ for every contribution is simply the distance between the line segment markings. The angle in the dot product is always $0^\circ$, since we are always orienting the black line along the path, so the above equation reduces to:

$$\mu_0 N I_{\text{PowerSupply}} = \oint B \cdot dl \approx \sum B \cdot \Delta l \cos(\theta) = \sum (B)(\Delta l)$$

Do not reverse the direction of circulation around the loop during your summation.

Caution: Keep Magnetic material (steel watch bands, bracelets, etc.) away from the experiment. Besides the solenoid, currents in power supplies, computers, etc. produce magnetic fields

Overview

In this experiment you will investigate the magnetic field produced by a current in a solenoid and in a flat, multi-turn coil. The purpose of the experiment is to provide some intuition about the spatial variation of the magnetic field produced by common current
distributions.

Since the magnetic field is a vector quantity, an instrument used to measure the field must be sensitive to both the magnitude and direction of the field. In this experiment we use both a magnetic compass and a *Hall Probe* to study the field. The compass can only indicate the direction of the field. The Hall probe generates an output voltage *proportional* to the component of the magnetic field perpendicular to its face, thus measuring the magnitude of the field.

In order to measure different components of the magnetic field, the Hall probe is hinged and can be rotated. The position of the probe for measuring the field along the axis of the flat coil is shown in Figure 2.

![The Hall probe measuring the field of the flat coil](Figure2.jpg)

The purpose of the part of the experiment with the solenoid (Figure 3) is to determine the coefficient of proportionality between the output of the Hall probe and the strength of the magnetic field.
Figure 3: The Solenoid

As shown in your textbook, at a point inside a long solenoid the field produced by a current $I$ is parallel to the axis and has magnitude:

$$ B = \mu_0 n I $$

(1)

Where $n$ is the number of turns per meter and $\mu_0$ is the magnetic permeability constant equivalent to $4\pi \times 10^{-7}$ N·A$^{-2}$. Since our solenoid is not long enough to be ideal, the magnetic field may not be constant everywhere. However, at points well inside the solenoid the B-field is nearly uniform (constant in magnitude and direction).

The field produced by a current in a flat coil is not as simple as the field inside a long solenoid. At a point on the $z$-axis of the coil in Figure 4 the field is:

$$ B = \frac{\mu_0 N r^2 I}{2(r^2 + z^2)^{3/2}} $$

(2)

where $I$ is the current in the coil, $r$ is the radius of the coil, $N$ is the number of turns, and $z$ is the distance from the observation point to the plane of the coil.
The map of the magnetic field of a flat coil is illustrated in Figure 5. The field lines always form closed curves, and the overall field distribution is similar to that of a permanent bar magnet with its North-South axis pointing along the z axis of the coil.

Figure 5: Magnetic Field of a Flat Coil

Procedure

Part A: Calibration of the Hall Probe Using an Air-Core Solenoid

Warning: Set the coil current to zero before changing coil connections.

1) Connect the solenoid to the power source.
2) Using the compass, make a map of the magnetic field lines of the solenoid.
3) Disconnect the Hall probe assembly from its mounting block and attach it to the end of the plastic meter stick. Be very careful with the plastic screw.

**Make sure the range is set to 6.4 mT on the Hall probe.**
Figure 6: The Hall Effect Probe placed inside the solenoid to measure the magnetic field.

4) Insert the Hall probe into the solenoid tube so that the probe is close to the middle of the solenoid (along the coil axis).

5) To measure the magnetic field with the probe, open Logger Pro 3.4.6/ Data Studio Software (as appropriate) in the Physics folder:

1. Click on the LabPro icon (sensor setup)
2. select analog CH1
3. choose the sensor (Voltage)
4. choose differential Voltage

6) To collect data

1. Press the Collect button
2. data will be collected for 10 seconds (the default)
3. click the STAT button to open a window
4. record the mean and STD DEV
5. do not close the STAT window or graph window (the STAT function only works with a Graph )

7) To make more measurements, Press Collect again. The STAT window will refresh as the data is collected.
8) Set the current to approximately 0.5 A using the multimeter, and record the coil current.

9) Increase the current in steps of 0.5 A up to a maximum of 2.5 A and record the currents and Hall probe readings. **2 points off if you blow the fuse!!**
10) Calculate the B field from the current values for each point using (1) and plot the calculated values versus the measured Hall voltages. This is your calibration line graph for the Hall Probe.

11) Using the calibration line graph (slope y) convert the measured Hall voltages to their corresponding magnetic field values (in Tesla) and check that they give you back the same values calculated in step 7) using (3). That is:

\[ B = V_{\text{Hall}} \cdot y \]  

**Part B: The Magnetic Field produced by a flat coil**

1) Connect the flat coil to the power source, and set the current to 1 A.

**Warning: do not exceed 1.5 A**

2) Use the compass to map the field lines of the coil. When you are done with the compass, please do not leave it close to the magnet!

3) Both the flat coil and the Hall probe with its mounting block are made to sit or slide on the meter stick as shown on Figure 2. Orient the Hall probe so that the arrow is perpendicular to the plane of the coil.

4) Record the Hall probe voltage at one centimeter intervals starting from the center of the coil \((z = 0)\) and moving along the z axis.

5) Convert the magnetic field at points on the z axis of the coil to a linear plot.

**Before you leave- make sure the knife switch is positioned straight up (OFF position)**

For the report- Make sure to include in your lab report the following items:

1) Maps of the B field for Permanent magnet, the Solenoid and the Coil.

2) **From part A:** A Plot of B vs \(V_{\text{Hall}}\) with error bars. Find the slope and use the Range/2 method to find the error in slope.

3) **From part B:** A Plot of B vs z.